

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

A Novel Form of πgb- Closed Sets in Extended Topological Spaces

S.Reena¹, F.Nirmala Irudayam²

P.G. Student, II M.Sc., Nirmala College for Women, Coimbatore, India

Associate Professor, Dept. of Mathematics, Nirmala College For Women, Coimbatore, India

ABSTRACT: In this paper we have introduced a new class of closed set in simple extended topological space called π - generalized b⁺ closed set (briefly π gb⁺-closed set) and some of its characteristics are investigated. Further we have intended to study of its applications.

KEYWORDS: π gb⁺-closed set, π gb⁺-closure operator, π gb⁺-open, π gb⁺- T_{1/2} space.

I. INTRODUCTION

Levine [18] and Andrijevic [1] introduced the concept of generalized open sets and b-open sets respectively in topological spaces. The class of b-open sets is contained in the class of semi pre-open sets and contains the class of semi-open and the class of pre-open sets. Since then several researches have proposed various notions of generalized closed sets as generalized semi-closed, generalized pre-closed and generalized semi pre-open sets respectively in [20, 3, 31]. In 1968 Zaitsev [33] defined π -closed sets. Later Dontchev and Noiri [7] introduced the notion of πg -closed sets. Park [26] defined πgp -closed sets. Then Aslim, Caksu and Noiri [4] studied the notion of πgs -closed sets. The idea of πgb -closed sets were introduced by D.Sreeja and S.Janaki [30]. Later the properties and characteristics of πgb -closed sets were introduced by Sinem Caglar and Gulhan Ashim [29]. In 1963, Levine [17] introduced the concept of simple extension of a topology τ by a non open set B as $\tau^+(B)=\{O\cup(O' \cap B) / O, O' \in \tau, B \notin \tau\}$. Eventually F. Nirmala and I.Arockiarani[22] proposed the idea of b open sets in simple extended topology and extended it to ideal topology in simple extension as well.

The aim of this paper is to coin a new type of b closed set in the light of simple extension topology and investigate its properties. In section 3 we study the basic properties of πgb^+ -closed sets and introduce idea of πgb^+ -T_{1/2} space and its properties are discussed. Finally in section 4, πgb^+ -open sets is defined and its properties are putforth.

In this paper, (X, τ^+) (or simply X) represent a non-empty extended topological space on which no separation axioms are assumed, unless and otherwise mentioned. Let A be a subset of a space X. Then the closure of A is assumed in the extended topology and the interior of A is taken in general topology are denoted by cl^+ (A) and int (A) respectively.

II. PRELIMINARIES

Definition 2.1:

A subset A of a topological space X is called a: (a) α -open set [23] if A \subset int (cl (int (A))) (b) semi-open set [17] if A \subset cl (int(A)) (c) pre-open set [21] if A \subset int(cl (A)) (d) semi-pre open set [1] if A \subset cl (int (cl (A))) (e) regular – open [32] if A = int (cl (A))



(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

(f) b – open [2] or γ – open [9] if A \subset int (cl (A)) U cl (int (A))

Definition 2. 2:

A subset A of a space (X, τ) is called π open [33] if A is a finite union of regular open sets.

Definition 2.3:

A subset A of a topological space X is called: (a) g - closed [18] if cl (A) \subset U whenever A \subset U and U is open in X (b) gp - closed [20] if pcl (A) \subset U whenever A \subset U and U is open in X (c) gs - closed [3] if scl (A) \subset U whenever A \subset U and U is open in X (d) gb - closed [14] if bcl (A) \subset U whenever A \subset U and U is open in X (e) π g - closed [7] if cl (A) \subset U whenever A \subset U and U is π - open in X (f) π gp - closed [26] if pcl (A) \subset U whenever A \subset U and U is π - open in X (g) π gs- closed [4] if scl (A) \subset U whenever A \subset U and U is π - open in X (g) π gs- closed [15] if α cl(A) \subset U whenever A \subset U and U is π - open in X (i) π gsp-closed [28] if spcl (A) \subset U whenever A \subset U and U is π - open in X (j) π gb - closed [30] if bcl (A) \subset U whenever A \subset U and U is π - open in X

Definition 2.4:

A subset A of a topological space X is said to be a:

(a) pre⁺-open set [22] if $A \subset int(cl^+(A))$

(b) semi⁺-open set [22] if $A \subset cl^+$ (int(A))

(c) α^+ -open set [22] if A \subset int (cl⁺ (int (A)))

(d) semi-pre⁺ open set [22]if $A \subset cl^+$ (int (cl^+ (A)))

(e) b^+ - open [22] if $A \subset int (cl^+(A)) \cup cl^+(int (A))$

(f) Q - set [19] if int (cl(A)) = cl(int (A)).

Definition 2.5:

A topological space X is said to

- (a) extremely disconnected [6] if the closure of every open subset of X is open in X.
- (b) hyper connected [5] if the closure of every open subset is X.
- (c) $\pi gb T_{1/2}$ space [18] if every πgb closed set is b closed.

III. πgb + - CLOSED SETS

Definition 3.1:

(a) A subset A of a topological space X is said to be regular – open in simple extension topology if $A = int (cl^+(A))$.

(b)A subset A of a space (X, τ^+) is called π^+ open if A is a finite union of regular⁺ open sets in simple extension topology.

The intersection of all b^+ - closed sets containing A is said to be b^+ - closure of A and is denoted by $b^+cl(A)$. Lemma 3.2:

Let A be a subset of a space X. Then

(a) $b^+cl(A) = s^+cl(A) \cap p^+cl(A) = A \cup ((int(cl^+A)) \cap cl^+(int(A))),$

(b) b^{+} int (A) $=s^{+}$ int (A) $\cup p^{+}$ int (A) $=A \cap (int (cl^{+} (A)) \cup cl^{+} (int (A))).$

Definition 3.3:

A subset A of a topological space X is called: (a) g^+ - closed if $cl^+(A) \subset U$ whenever $A \subset U$ and U is open in X (b) gp^+ - closed if $p^+cl(A) \subset U$ whenever $A \subset U$ and U is open in X



(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

(c) gs^+ - closed if s^+ cl (A) \subset U whenever A \subset U and U is open in X

(d) gb^+ - closed if b^+ cl (A) $\subset U$ whenever A $\subset U$ and U is open in X

(e) πg^+ - closed if cl⁺ (A) \subset U whenever A \subset U and U is π^+ - open in X

(f) $\pi g p^+$ - closed if $p^+ cl$ (A) $\subset U$ whenever A $\subset U$ and U is π^+ - open in X

(g) π gs⁺- closed if s⁺cl (A) \subset U whenever A \subset U and U is π ⁺ - open in X

(h) $\pi g \alpha^+$ -closed if α^+ cl(A) $\subset U$ whenever A $\subset U$ and U is π^+ -open in X

(i) π gsp⁺-closed if sp⁺cl (A) \subset U whenever A \subset U and U is π^+ -open in X.

The family of all πgb^+ - closed (resp. b⁺ - closed) sets in a topological space (X, τ^+) is denoted by $\pi GB^+C(X)$ (resp. $B^+C(X)$).

Definition 3.4:

A subset A of (X, τ^+) is called πgb^+ - closed if $b^+cl(A) \subset U$ whenever $A \subset U$ and U is π^+ - open in (X, τ^+) .

Theorem 3.5:

Every closed set is πgb^+ - closed

Proof:

Let A be any closed set in X such that $A \subseteq U$, where U is π^+ open. We know that $b^+cl(A) \subseteq cl(A)$ and since A is closed $b^+ cl(A) \subset cl(A) = A$. i.e. $b^+ cl(A) \subset A$ and $A \subset U$, where U is π^+ open . Therefore \mathbf{b}^+ $cl(A) \subset U$. Hence A is πgb^+ - closed set in X.

Theorem 3.6:

- a. Every g^+ -closed is $\pi g b^+$ closed
- b. Every α^+ closed set is πgb^+ closed c. Every pre⁺ closed set is πgb^+ closed
- d. Every gb^+ -closed set is πgb^+ closed

- e. Every π^+ closed set is πgb^+ -closed. f. Every πg^+ closed set is πgb^+ -closed. g. Every πgp^+ closed set is πgb^+ closed h. Every πga^+ closed set is πgb^+ closed.
- i. Every πgs^+ closed set is πgb^+ closed.
- j. Every πgb^+ closed set is πgsp^+ closed.

Proof:

Obvious, follows from the definition .

Converses of the above need not be true as seen in the following example.

Example 3.7:

Consider X={a,b,c}, $\tau = \{\emptyset, \{b\}, X\}$. Let B ={c} then $\tau + = \{\emptyset, \{b\}, \{c\}, \{b,c\}, X\}$. Let A={b}. Then A is $\pi g b^+$ -closed but not closed, g^+ -closed, α^+ -closed, gb^+ -closed, πga^+ -closed, $\pi g\alpha^+$ -closed, pre^+ -closed, πgp^+ -closed.



(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

Remark 3.8:



1 $\pi \sigma b^+$ -closed set	6 ga^+ -closed set	11 pre ⁺ -closed set	16 πq^+ -closed set
2. alaradaat	7. gu -closed set	11. pre -closed set 12 mm^{+} -closed set	$10. \text{ mg}^{-}$ erosed set
2. closed set	7. semi closed set	12. gp -closed set	$1/. \pi g \alpha$ -closed set
3. g^+ -closed set	8. sp^+ -closed set	13. gsp^+ -closed set	18. πgp^+ -closed set
4. α^+ -closed set	9. sg^+ -closed set	14. b^+ -closed set	19. πgsp^+ -closed set
5. α^+ g -closed set	10. gs^+ -closed	15. gb^+ -closed set	

Theorem 3.9:

If A is π^+ -open and πgb^+ - closed, then A is b^+ -closed. **Proof:**

Let A is π^+ -open and πgb^+ -closed. Let A \subseteq A where A is π^+ -open. Since A is πgb^+ - closed, $b^+ cl (A) \subseteq A$. Then A = b⁺ cl (A).Hence A is b⁺-closed.

Theorem 3.10:

For a subset A of X, the following statements are equivalent:

(1) A is π^+ - open and πgb^+ - closed.

(2) A is regular+ open.

Proof:

(1) \Rightarrow (2) Let A be a π^+ - open and πgb^+ - closed subset of X. Then $b^+ cl (A) \subset A$ and so int $(cl^+ (A)) \subset A$ holds. Since A is open then A is pre⁺ - open and thus A \subset int $(cl^+ (A))$. Therefore, we have int $(cl^+ (A)) = A$, which shows that A is regular⁺ open.

(2) \Rightarrow (1) Since every regular⁺ open set is π^+ - open then b⁺ cl(A)=A and b⁺ cl(A) \subset A. Hence A is πgb^+ - closed.

Definition 3.11:

A subset A of a topological space X is said to be Q^+ - set if int $(cl^+ (A)) = cl^+(int (A))$.



(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

Theorem 3.12:

For a subset A of X, the following statements are equivalent: (1) A is π^+ - clopen, (2) A is π^+ - open, Q⁺ - set and πgb^+ - closed.

Proof:

(1) \Rightarrow (2) Let A be a π^+ - clopen subset of X. Then A is π^+ - closed and π^+ - open. Thus A is closed and open. Therefore, A is Q^+ - set. Since every π^+ - closed is $\pi g b^+$ - closed then A is $\pi g b^+$ - closed.

(2) \Rightarrow (1) By Theorem 3.10, A is regular⁺ open. Since A is Q⁺ - set, A = int (cl⁺(A)) = cl⁺(int(A)). Therefore, A is regular+ closed. Then A is π^+ - closed. Hence A is π^+ - clopen.

Theorem 3.13:

Let A be πgb^+ -closed in(X, τ^+). Then b^+ cl (A)-A does not contain any non empty π^+ closed set.

Proof:

Let F be a non empty π^+ -closed set such that $F \subset b^+ cl$ (A)-A. Since A is πgb^+ -closed, $A \subset X$ -F where X-F is open implies b⁺ cl (A) \subset X-F. Hence F \subset X-b⁺ cl (A). Now, F \subset b⁺ cl (A) \cap (X-b⁺ cl (A)) implies F= Ø which is a contradiction. Therefore b^+ cl (A) does not contain any non empty π^+ - closed set.

Corollary 3.14:

Let A be πgb^+ -closed in (X, τ^+) . Then A is b^+ -closed if $b^+ cl(A)$ -A is π^+ closed. **Proof:** Obvious.

Theorem 3.15:

If A is πgb^+ -closed set and B is any set such that $A \subseteq B \subseteq b^+ cl$ (A), then B is πgb^+ -closed set.

Proof:

Let B \subset U and U be π^+ -open. Given A \subset B. Then A \subset U. Since A is π gb⁺- closed, A \subset U implies b⁺cl (A) \subset U. By assumption it follows that $b^+ cl(B) \subset b^+ cl(A) \subset U$. Hence B is a πgb^+ -closed set.

Remark 3.16:

Finite union of πgb^+ -closed sets need not be πgb^+ -closed.

Example 3.17:

Consider X= {a, b, c}, $\tau = \{\emptyset, \{a, b\}, X\}$. Let B = {c} then $\tau^+ = \{\emptyset, \{c\}, \{a, b\}, X\}$ Let A = {a}, B = {b}. Here A and B are πgb^+ - closed but A $\cup B = \{a, b\}$ is not πgb^+ -closed.

Remark 3.18:

Finite intersection of πgb^+ -closed sets need not be πgb^+ - closed.

Example 3.19:

Consider X = {a, b, c}, $\tau = \{\emptyset, \{b\}, \{b, c\}, X\}$. Let B = {c} then $\tau^+ = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}$ Let A = {a,b}, В = {b,c}. Here A and B are πgb^+ -closed but A \cap B = {b} is not πgb^+ -closed.

Proposition 3.20:

Let A be a subset of a topological space X. If A is semi⁺-open then $p^+cl(A) = cl^+(A)$.

A topological space X is said to be extremely disconnected in τ^+ if the closure in the extended topology of every open subset of X is open in X.

Theorem 3.21:

A space X is extremely disconnected in τ^+ if and only if every πgb^+ - closed subset of X is πgp^+ - closed.



(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

Proof:

Suppose that X is extremely disconnected. Let A be πgb^+ - closed and let U be a π^+ open set containing A. Then $b^+cl(A) = A \cup [int (cl^+(A) \cap cl^+ (int(A))] \subset U$, i.e. $[int(cl^+(A)) \cap cl^+ (int(A))] \subset U$. Since $int(cl^+(A))$ is closed, we have $cl^+ (int(A)) \subset cl^+ [int(cl^+(A)) \cap int(A)] \subset [cl^+ (int(cl^+(A))) \cap cl^+ (int(A))] \subset U$. It follows that $p^+cl(A) = A \cup cl^+ (int(A)) \subset U$. Hence A is πgp^+ - closed.

To prove the converse, let every πgb^+ - closed subset of X be πgp^+ - closed. Let A be a regular⁺ open subset of X. Then $b^+cl(A) = A \cup [int(cl^+(A)) \cap cl^+(int(A))] = A \cup [A \cap cl^+(int(A))] \subset A$. Then A is πgb^+ - closed and so A is πgp^+ - closed. Since every regular⁺ open is semi⁺-open set and by Proposition 3.20, we have $cl^+(A) = p^+cl(A)$. Hence $cl^+(A) \subset A$. Therefore, A is closed. This shows that X is extremely disconnected.

A topological space X is said to be hyper connected in τ^+ if the closure in the extended topology of every open subset is X.

Theorem 3.22:

Let X be a hyper connected space. Then every πgb^+ - closed subset of X is πgs^+ - closed. **Proof:**

Assume that X is hyper connected. Let A be πgb^+ - closed and let U be a π^+ - open set containing A. Then $b^+cl(A) = A \cup [int(cl^+(A)) \cap cl^+(int(A))] = A \cup int(cl^+(A)) = s^+cl(A)$. Since $b^+cl(A) = s^+cl(A)$, we have $s^+cl(A) \subset U$. Hence, A is πgs^+ - closed.

Definition 3.23:

A topological space X is said to be πgb^+ -T_{1/2} space if every πgb^+ - closed set is b^+ - closed.

Theorem 3.24:

For a space X, the following statements are equivalent:

(1) X is $\pi gb^+ - T_{1/2}$,

(2) For every subset A of X, A is πgb^+ -open if and only if A is b^+ -open.

Proof:

(1) \Rightarrow (2) Let the space X be πgb^+ - T_{1/2} and let A be a πgb^+ -open subset of X. Then X \ A is πgb^+ -closed and so X \ A is b^+ -closed. Hence A is b^+ -open.

Conversely, let A be a b⁺-open subset of X. Thus $X \setminus A$ is b⁺-closed. Since every b⁺-closed set is πgb^+ -closed then $X \setminus A$ is πgb^+ -closed. Therefore, A is πgb^+ -open.

(2) \Rightarrow (1) Let A be a πgb^+ -open subset of X. Then X \ A is πgb^+ -open. By the hypothesis X \ A is b^+ -open. Thus A is b^+ -closed. Since every πgb^+ -closed set is b^+ -closed, thus X is πgb^+ -T_{1/2}.

IV. πgb^+ - OPEN SETS

Definition 4.1: A set $A \subset X$ is called πgb^+ -open if and only if its complement is πgb^+ - closed.

Remark 4.2: $b^+cl (X-A) = X-b^+int (A)$

Theorem 4.3:

If $A \subset X$ is πgb^+ -open iff $F \subset b^+$ int(A) whenever F is π^+ -closed and $F \subset A$.

Theorem 4.4:

If b^+ int (A) $\subset B \subset A$ and A is πgb^+ -open then B is πgb^+ -open.

Remark 4.5: For any $A \subset X$, b^+ int $(b^+ cl(A) - A) = \emptyset$.



(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

Theorem 4.6:

If $A \subset X$ is πgb^+ - closed, then $b^+cl(A)$ -A is πgb^+ - open.

Theorem 4.7:

(i) $B^+O(\tau^+) \subset \pi GB^+O(\tau^+)$ (ii) A space (X, τ^+) is $\pi gb^+ - T_{1/2}$ iff $B^+O(\tau^+) = \pi GB^+O(\tau^+)$. **Proof:** (i) Let A be b^+ -open, then X-A is b^+ -closed so X-A is πgb^+ -closed. Thus A is πgb^+ -open. Hence $B^+O(\tau^+) \subset \pi GB^+O(\tau^+)$ (ii) **Necessity:**

Let (X, τ^+) be πgb^+ - T_{1/2} space. Let $A \in \pi GB^+O(\tau^+)$. Then X-A is πgb^+ - closed. By hypothesis, X-A is b^+ -closed thus $A \in B^+O(\tau^+)$. Thus $\pi GB^+O(\tau^+) = B^+O(\tau^+)$.

Sufficiency:

Let $B^+O(\tau^+) = \pi \ GB^+O(\tau^+)$. Let A be πgb^+ -closed. Then X-A is πgb^+ - open. X-A $\in \pi \ GB^+O(\tau^+)$. X-A $\in B^+O(\tau^+)$. Hence A is b⁺-closed. This implies (X, τ^+) is $\pi gb^+ - T_{1/2}$ space.

V. CONCLUSION

In this paper we have introduced new closed sets in simple extension topology through which its corresponding open sets is defined .As an application of these sets we have introduced new types of spaces which would open new areas of further study.

REFERENCES

- [1] D. Andrijevic, Semi preopen sets, Mat. Vesnik 38 (1986), 24-32.
- [2] D. Andrijevic, On b-open sets, Mat. Vesnik 48 (1996), 59-64.
- [3] S. P. Arya and T. M. Nour, Characterizations of s-normal spaces, Indian J. Pure Appl. Math.21 (1990), no. 8, 717-719.
- [4] Aslim G., Caksu Guler A. and Noiri T., On πgs-closed sets in topological spaces, Acta Math. Hungar. 112(4) (2006),275-283.
- [5] Benchalli S.S. and Bansali P. M., gb-Compactness and gb Connectedness, Int. J.Contemp. Math. Sciences, Vol. 6, 2011, no:10, 465-475.
- [6] Bourbaki N., General Topology, Part I. Mass. : Addison Wesley Reading; 1966.
- [7] Dontchev J. and Noiri T., Quasi normal spaces and πg-closed sets, Acta Math. Hungar., 89 (2000), 211-219.
- [8] Dontchev J. and Ganster M., On δ-generalized closed sets and T3/4 -spaces, Mem. Fac. Sci. Kochi Univ. Ser. A (Math.), 17(1996), 15-31
- [9] E. Ekici and M. Caldas, Slightly -continuous functions, Bol. Soc. Parana. Mat. (3) 22 (2004), 63-74.
- [10] Ekici E., On γ-normal spaces, Bull. Math. Soc. Sci. Math. Roumanie Tome 50 (98) No.3, 2007, 259-272.
- [11] Ekici E. and Baker C. W., On πg-closed sets and continuity, Kochi J. Math. 2 (2007), 35-42.
- [12] Ekici E.and Caldas M., Slightly γ-continuous functions, Bol. Soc. Parana. Mat. (3) 22 (2004), no.2, 63-74.
- [13] El-Atik A. A., A study on some types of mappings on topological spaces, M. Sc. Thesis, Egypt: Tanta University, 1997.
- [14] Ganster M. and Steiner M., On br-closed sets, Appl. Gen. Topol. 8 (2007), no.2, 243-247.
- [15] C.Janaki, Studies on g-closed sets in Topology, Ph.D Thesis, Bharathiar University, Coimbatore. (2009).
- [16] Keskin A., Noiri T., Almost b-continuous functions, Chaos, Solitons and Fractals (2007)
- [17] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math.Monthly 70(1963), 36-41.
- [18] N. Levine, Generalized closed sets in topology, Rend . Circ. Mat. Palermo (2) 19 (1970),89-96.
- [19] Levine N., On the commutivity of the closure and interior operator in the topological spaces, Amer. Math. Monthly, 68 (1961), 474-477.
- [20] H. Maki, J. Umehara and T. Noiri, Every topological space is pre-T1/2, Mem. Fac. Sci. Kochi Univ. Ser. A Math. 17 (1996), 33-42.
- [21] Mashhour A. S., Abd El-Monsef M. E. and El-Deep S. N., On pre continuous and weak pre continuous mappings, Proc. Math. Phys. Soc. Egypt No.53 (1982), 47-53.
- [22] F.Nirmala Irudayam and Sr.I.Arockiarani, A note on the weaker form of bI sets and its generalization in SEITS ,International Journal of computer application, Issue 2,vol 4 (Aug 2012),41-54.
- [23] O. Njastad, On some classes of nearly open sets, Pacific J. Math. 15 (1965), 961-970.
- [24] Noiri T., Mildly normal spaces and some functions, Kyungpook Math. J., 36 (1996), 183-190.
- [25] Omari A. A. and Noorani M. S. M., On generalized b-closed sets, Bull. Malays. Math. Sci. Soc. (2) 32 (1) (2009), 19-30.
- [26] Park J. H., On πgp-closed sets in topological spaces, Indian J. Pure Appl. Math. (2004).
- [27] Park J. H and Park J. K., On πgp-continuous functions in topological spaces, Chaos, Soliton and Fractals 20 (2004) 467-477.
- [28] M. S. Sarsak and N. Rajesh, -Generalized Semi-Preclosed Sets, International Mathematical Forum, 5 (2010), 573-578.
- [29] Sinem Cagler Akgun and Gulhan Aslim, "On *π*gb-closed sets and Related Topics.(2012)



(An ISO 3297: 2007 Certified Organization)

Vol. 5, Issue 5, May 2016

- [30] Sreeja D.and Janaki C., On *π*gb-closed sets in topological spaces, International Journal of Mathematical Archieve-2 (8), 2011, 1314-1320.
- [31] L.A. Steen and J.A. Seebach, jr. Counter examples in topology, Holt, Rinehart and
 [32] Winston, Inc., U.S.A., 1970.
- [33] Stone M. H., Applications of the theory of Boolean rings to general topology, Trans. Am. Math. Soc. 1937; 41: 375-81.
- [34] Zaitsev V., "On certain class of Topological spaces and their bicompactification", Dokl. Akad. Nauk SSSR 178 (1968), pp.778-779.